DYNAMICS-WIN:

a numerical toolkit
for exploring and analyzing the dynamics
of low dimensional systems of
differential and difference equations

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1. INTRODUCTION to DYNAMICS-WIN

The program Dynamics-Win

The computer program Dynamics-Win is for the novice and the expert, and helps the novice begin immediately exploring dynamical systems with a broad array of interactive techniques. When investigating a dynamical system, it is of course important to be able to plot trajectories, but that is just the beginning. An array of tools have been developed and combined in a single software package to help visualize what is happening in dynamical systems. Many of these tools such as plotting of "basins of attraction", computing "straddle trajectories", and automatically searching for all periodic orbits of a specified period are in this program Dynamics. These tools are elementary in that what they do can be understood by undergraduates and they are fundamental. The algorithms to implement these ideas are sometimes fairly sophisticated. Often, we refer to Dynamics-Win simply as Dynamics.

Running Dynamics

The program Dynamics calls upon the files y.pic, yprom.txt, ymenus.txt, yps.txt, yhelp.txt, and yalert.txt. They should be in the active directory when you start the program Dynamics. For example, if you are going to run the program from the directory C:\DynWin, use the command "cd \DynWin" in the command line to make that the active directory. If you want to add a map or differential equation, then the file gserver.drv is needed.
1.1 GETTING STARTED WITH DYNAMICS

To get started, if the directory that contains the program Dynamics-Win is C:\DynWin, for example, then type from the Run command line the command

C:\dynwin\dynamics-win <Enter>

It may be handy to activate the directory by typing in the command line "cd C:\DynWin <Enter> " and then Run dynamics-win <Enter>. You also may run the program just by clicking the appropriate folders/files via my computer.

First, the title page of Dynamics appears on the screen for a few seconds and then the PROcess Menu. This menu will include the following

MAP MENU 1
DE - Differential Equations menu
OWN - enter your OWN process

MAPS
C - complex Cubic map
CW - CobWeb map, a 1-dim map
H - Henon map
LOG - LOGistic map
TT - TenT map

After selecting the Differential Equations menu, hit <Enter> and the differential equations of the process menu appear on the screen and include

DIFFERENTIAL EQUATIONS MENU
D - forced double-well Duffing equation
GN - GoodwiN equation
L - Lorenz system
LV - Lotka/Volterra equations
P - forced damped Pendulum equation
VP - forced Van der Pol equation

To select the Henon map (H) in the initial process menu (map menu 1), either use the arrow keys to select this process and hit <Enter> or type (both upper case and lower case letters may be used) H<Enter>

The Main Menu appears on the screen

NumExplM - FileM - ParameterM - VectorM - ScreenM - Help

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Select NumExplM and hit <Enter> or type nem<Enter> and the Numerical Explorations Menu appears and includes

<table>
<thead>
<tr>
<th>NEM</th>
<th>-- NUMERICAL EXPLORATIONS MENU</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>plot Trajectory</td>
</tr>
<tr>
<td>DYN</td>
<td>quit &amp; start new map or Differential Eqn.</td>
</tr>
<tr>
<td>OWN</td>
<td>quit &amp; create OWN process</td>
</tr>
</tbody>
</table>

MENUS

BIFM  - BIFurcation diagram Menu
BM    - Basin of attraction Menu
STM   - Straddle Trajectory Menu
UM    - Unstable and stable manifold Menu

If you are in the numerical explorations menu, either hit <Esc> or type mm<Enter> to return to the main menu. Use the arrow keys to select FileM or type fm<Enter> and the File Menu appears and includes

<table>
<thead>
<tr>
<th>FM</th>
<th>-- FILE MENU</th>
</tr>
</thead>
<tbody>
<tr>
<td>FD</td>
<td>retrieve picture (From Disk)</td>
</tr>
<tr>
<td>PP</td>
<td>Print Picture</td>
</tr>
<tr>
<td>TD</td>
<td>save picture (To Disk)</td>
</tr>
</tbody>
</table>

MENUS

SCM - Size of Core picture Menu
DM  - Disk files Menu

If you are in the file menu, either hit <Esc> or type mm<Enter> to return to the main menu. Use the arrow keys to select Help and hit <Enter> or type h<Enter> and the help menu appears on the screen. There are two windows in the help menu. The top window contains the commands and the bottom window contains information. Many menus do have this structure. The bottom window is referred to as the information window.

Quick start tutorial

If you use the arrow keys to select the command tut and hit <Enter>, you will be presented with a quick start tutorial. This tutorial deals with the simplest commands available in Dynamics and yet provide a view of the capabilities of the program. Dynamics provides a wide range of capabilities to the experienced user and yet is rather easy for the novice to use.

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Enter a menu item or enter a command

There are two ways to select the menu item H. One approach is to use the "point and shoot" menu system. When a menu appears, the bottom line of the box that contains the menu presents an item from the menu. That item is highlighted and enclosed by two asterisks. By hitting the arrow key, you can move from item to item. When the one you want is selected, hit <Enter> to implement this selection. You also can click on a command with the left mouse button to high-light an item. When the one you want is selected, click the left mouse button again to implement this selection.

The second approach allows you to ignore which menu, if any, is on the screen. When you remember the command, this approach may be easier. Just type the command and hit <Enter>. When typing a command, you can use either upper or lower case letters.

For purposes of exposition, we emphasize the latter approach, but you may choose occasionally to use the "point and shoot" approach. The phrase "enter a command" means type the command (or select the command from a menu on the screen using the arrow keys) and then hit the <Enter> key.

Retrieving menus

Start the program and select the Henon map. Now the main menu, a list of menus, appears on the screen. Once you have selected a process and are working with it, if no menu is on the screen, you can fetch the previous menu by hitting <Enter> (assuming you are not in the midst of typing a command). If a menu is on the screen, fetch its parent menu by hitting <Esc>. This <Esc> can be repeated until the main menu appears. You can go directly to the main menu by entering command mm.

Viewing the latest menu while plotting

Sometimes you want to see the last-called menu while the computer is plotting. If the computer is plotting, then whenever you hit <Enter>, the program pauses and the last called menu appears on the screen. In case you do not want to have this latest menu, just hit <space bar> and the menu is erased and the computer continues plotting.

Help with the commands

To get help with the command that is highlighted in a menu on the screen, just hit the <Tab> key. ("Hit" a key means gently depress and release the key.) For most commands, a description will appear on the screen. There is another way to get this information and can be used even if the command is not showing on the screen. To get help with a command, hit <*> and enter the command. For example, to see what command b does, type *b<Enter> and the program prints out the description. (Notice there is no space between * and b.)
1.2 EXAMPLE: entering commands

Let $F$ be a continuous map from the $n$-dimensional phase space to itself and consider the corresponding discrete time process $x_{k+1} = F(x_k)$. For each point $x_0$, the point $x_k$ is called the $k$th iterate of the point $x_0$.

The trajectory of any point $x$ in the phase space is the finite or infinite sequence of consecutive iterates of $x$.

Given a pair $(x,y)$, applying the Henon map once gives a new pair using the formula $(\rho - x^2 + c_1 y, x)$. The symbol `$\star$' in processes of Dynamics (maps and differential equations) denotes multiplication. The Jacobian determinant of the Henon map is $-c_1$.

Sometimes, for example, when plotting a trajectory, there are two $<$Enter$>$ keys "$\leftarrow\leftarrow$" because after the first $\leftarrow$ a menu of hints and options appears. By hitting the second $\leftarrow$ you are ignoring these options instructing the program that you do not want to use these options.

**dynamics-win $\leftarrow$** Start the program Dynamics-Win;

**h $\leftarrow$** select the Henon map and the main menu appears;

**nem $\leftarrow$** get the numerical explorations menu;

**plotting a trajectory**

t $\leftarrow$ $\leftarrow$ plot the trajectory (initial condition $y$)

The initial values for $x$ and $y$ are 0.0 and 2.0 respectively, and the program will apply the process (which is the Henon map) to the point $(x,y)$ and then plot the resulting point. In other words, the program replaces the value of $y = (x,y)$ with the value $(\rho - x^2 + c_1 y, x)$, and it will do this repeatedly, that is, it "iterates" the map. A sequence of dots will appear in rapid succession, hundreds or thousands per second.

**drawing a box**

The plotting of the trajectory can be paused at any time in order to perform other commands that enhance or change what the program is doing. For example, you can draw a box around the plotted trajectory by locating the "Box" command in the menu system and executing it.

$\leftarrow$ return to numerical explorations menu;

$<\text{Esc}>$ get the main menu on the screen;

**sm $\leftarrow$** select the screen menu;

**bxm $\leftarrow$** select the box menu;

**b $\leftarrow$** draw a box;

and a box will be drawn around the screen which frames the trajectory.
Figure 1-1: Trajectory of the Henon map

This picture shows a trajectory of the Henon map (b)

\[(x, y) \rightarrow (1.4 - x^2 + 0.3y, x)\]

In fact, the trajectory of any initial conditions selected from some open set yields a similar picture if the first 20 iterates or so are not plotted. Let \(F: \mathbb{R}^2 \rightarrow \mathbb{R}^2\) be a continuous map. A compact set \(S \subset \mathbb{R}^2\) is called an attractor for \(F\) if (a) \(F(S) = S\); (b) there exist an open neighborhood \(U\) of \(S\) such that for every \(x \in U\) the distance of \(x_n\) and \(S\) converges to zero as \(n \rightarrow \infty\) (more precisely, for every \(\varepsilon > 0\), there exists an positive integer \(N\) such that for every \(n \geq N\) the distance of \(x_n\) and \(S\) is less than \(\varepsilon\)); (c) the collection of limit points of \(\{x_n\}\) is equal to \(S\). In fact, the picture shows an attractor. This attractor is known as the Henon attractor. Frequently, this attractor is called a "strange attractor" because it appears to be a Cantor-like set of curves. Henon may have chosen \(\text{rho} = 1.4\) and \(c_1 = 0.3\) because of the beauty of the attractor. What do the attractors look like when the Jacobian \(c_1\) is chosen larger than 0.3 for various rho values? (There are no attractors if \(|c_1| > 1\).)

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viewing the parameter menu

<Esc> terminate the plotting of the trajectory
and the box menu appears (since it was the last menu you called). Hit the
key <Esc> twice more and the main menu appears on the screen.

pm ← get the parameter menu to view the parameter settings

The program gives rho the default parameter value of 2.12 and ct the value
-0.3. For this process, the initial setting for the screen is that x is
plotted horizontally with a scale running from -2.5 to 2.5, and y is
plotted vertically with the same scale.

<Esc> return to the main menu.

nem ← get the numerical explorations menu;

t ← ← ← plot the trajectory

and the previous picture returns to the screen and the program continues
plotting the trajectory. The picture can be returned to the screen because
the program has a separate copy of the picture in core memory, the "core"
copy of the picture, in addition to the screen picture. The core copy of
the picture usually has higher resolution than the screen. The core copy is
720 dots wide by 720 dots high if there is sufficient memory available, and
when the picture is sent to the printer, this higher resolution picture is
transmitted. The core picture does not include the words on the screen so
pictures can be refreshed (that is, cleaned up) using the command r.

refresh the screen and continue plotting

sm ← call the screen menu;

r ← refresh the screen;

to get a refreshed picture without all the text that was also on the
screen. Hence, the pictures sent to the printer are not muddied up with
extraneous text such as menus.

clear the screen and continue plotting

Assuming that a trajectory of the Henon map is still being plotted and
that a box has been drawn around the screen,

← ← ← return to screen menu;

c ← clear the screen and core memory;

The screen clears and the computer continues plotting the trajectory from
where it left off.

single stepping through a trajectory

Sometimes the computer plots too fast for what you want to see or do.
To see the process iterated at a slower rate, hit <.> and the process will
pause. Hit <.> again and the program will iterate the map once and plot the

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resulting point. A large cross is plotted at the current \((x,y)\) point. Repeatedly hitting \(< \cdot >\) you will see how the trajectory moves through the chaotic attractor. Hence, these processes can be walked through as slowly as you want.

\(c \leftarrow \) clear the screen and core memory, and immediately \(< \cdot > \ldots < \cdot >\) hit the period key \(< \cdot >\) several times and you will see the process single step, producing one dot for each \(\cdot\), and pausing thereafter. While the program is in this "pause after one dot" mode, you can still enter commands to reinitialize or clear, etc., and while the program is paused, a cross will appear on the screen at the current \((x,y)\) point.

\(<\text{space bar}>\) terminate single-stepping pause mode and the trajectory resumes iterating again as fast as it can.

**plot a cross at current position**

\(k \leftarrow\) plot a cross at current position and the computer will show the position of each dot it plots with a cross. This command is in the kruis menu \texttt{km}, which is in the screen menu \texttt{sm}. This feature is a toggle. A **toggle** is a command that turns some feature on or off. If it is on, entering the command turns it off and vice versa. Hence, when the command \(k\) is entered again, the feature turns off and no crosses are plotted anymore.

\(k \leftarrow\) turn the feature of plotting a cross off;

\(c \leftarrow\) clear the screen and core memory;

**initializing**

The current state of the trajectory is a vector called \(y\) (or sometimes called \(y_0\)). When the trajectory is paused, a large cross appears at that point. Another vector, the "initialization" vector, denoted \(y_1\), has certain special uses. When the trajectory is paused (or when the vector \(y_1\) is changed using the arrow keys) a small cross appears at its location. It is the initial condition for \(y\).

\(i \leftarrow\) initialize which replaces \(y\) by the position of \(y_1\). After the command \(i\) has been entered, the trajectory will be restarted from the small cross whose position is given by the initialization vector \(y_1\). The command \(i\) is in the kruis (cross) menu \texttt{km}, which is in the screen menu.

*Note. Using a mouse, you may wish to select a new initial condition by pointing the arrow at the desired position and double click the left button for initialization.*
viewing the y vectors

The values of the coordinates of the vectors y and y1 and some of the other storage vectors can be seen by entering the command yv.

\texttt{yv} \leftarrow \text{get the y vectors to view the storage vectors}
\texttt{r} \leftarrow \text{refresh the screen;}
to get rid of the text

\textbf{set storage vector y1 and initialize}

\texttt{<Esc>} \quad \text{terminate the plotting of the trajectory;}
\texttt{rho} \leftarrow 1.4 \leftarrow \text{set rho to be 1.4;}
\texttt{c1} \leftarrow 0.3 \leftarrow \text{set c1 to be 0.3;}
i \leftarrow \quad \text{initialize;}
t \leftarrow \quad \text{plot the trajectory}
and a warning "trajectory is too far from the screen: process paused" appears on the screen with a menu (recover menu) of possible corrections.
Now set y1 to be (0,0).
\texttt{sv} \leftarrow \quad \text{set vector;}
\texttt{sv1} \leftarrow \quad \text{set vector y1;}
\texttt{sv10} \leftarrow 0 \leftarrow \text{select sv10 to set coordinate \#0 of y1 equal to 0;}
\texttt{sv11} \leftarrow 0 \leftarrow \text{select sv11 to set coordinate \#1 of y1 equal to 0;}
Notice that if you are using the menu, you should now respond with \texttt{ok} twice by hitting \texttt{<Enter>} twice. Now initialize, and plot a trajectory.
i \leftarrow \quad \text{initialize;}
t \leftarrow \quad \text{plot the trajectory}
and (if appropriate) hit \texttt{<space bar>} to unpause the routine.

\textbf{change x scale or y scale}

In this example, the horizontal or x scale runs from -2.5 to 2.5. We want to change this scale to run from -2 to 2 using command xs.
\texttt{pm} \leftarrow \quad \text{fetch the parameter menu;}
\texttt{x5} \leftarrow -2 \quad 2 \leftarrow \text{change the x scale to run from -2 to 2;}
with a space between -2 and 2. You can similar change the y scale to run from -2 to 2 using the command ys.
\texttt{ys} \leftarrow -2 \quad 2 \leftarrow \text{change the y scale to run from -2 to 2;}
\texttt{e} \leftarrow \quad \text{clear the screen and core memory.}

\textit{quit Dynamics}

There two ways to quit the program. (a) You may quit \textit{Dynamics} such that an options menu appears (command q). For example, you may want to save some data or picture before exiting \textit{Dynamics}. (b) You may quit and exit \textit{Dynamics} without saving anything (command qx).
1.3 SOME BASIC COMMANDS

Basic commands for the screen (in screen menu sm)

<table>
<thead>
<tr>
<th>Command</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>Clear current window</td>
</tr>
<tr>
<td>CS</td>
<td>Clear Screen (but not core copy)</td>
</tr>
<tr>
<td>R</td>
<td>Refresh screen</td>
</tr>
</tbody>
</table>

C, CS
The command c clears the active window (and the core copy). The command cs clears the screen but not the core copy. The refresh command r will restore the picture.

R
The program keeps a copy of the picture in core that is or can be of higher resolution than the copy on the screen. The command r refreshes the screen, eliminating extraneous text by retrieving it from the core memory copy. The "core" memory copy is the copy of the picture that is used for printing high resolution copies on a printer.

Basic commands for drawing boxes (in box menu bxm)

<table>
<thead>
<tr>
<th>Command</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>B</td>
<td>draw Box</td>
</tr>
<tr>
<td>B1</td>
<td>draw Box with tic marks</td>
</tr>
<tr>
<td>B2</td>
<td>draw Box with double tic marks</td>
</tr>
<tr>
<td>BB</td>
<td>set Box equal to entire window</td>
</tr>
</tbody>
</table>

B, BB
The command b causes a box to be drawn. The default position of the box is the entire screen.
If you enter b and no box is drawn, this usually suggests the box is outside the screen area, due to your change in coordinates. Use bb to reset the box to be the whole screen, and try b again. When you enter command bb, no box is drawn, but if you then enter command b, a box will be drawn.

B1, B2
Command b1 draws a box and then draws tic marks, from 4 to 9 tic marks on a side, choosing the number so as to give natural divisions.
Command b2 draws a box and then draws a double set of tic marks. The first set is identical with the tic marks of command b1. The second set is a finer set of small tic marks.

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Basic commands for initializing and drawing crosses (in kruis menu km)

<table>
<thead>
<tr>
<th>Command</th>
<th>Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>Initialize y using y1</td>
</tr>
<tr>
<td>K</td>
<td>big cross along trajectory: OFF</td>
</tr>
<tr>
<td>KK</td>
<td>permanent cross at y</td>
</tr>
<tr>
<td>KK1</td>
<td>permanent cross at y1</td>
</tr>
</tbody>
</table>

Arrow keys move small cross which is at y1 or 2-click left mouse button at desired spot.

I
The command i reinitializes the trajectory vector y to be the current position of y1; the latter's position is shown by the small cross.

K
The command k results in a cross being plotted as each dot is plotted; the cross is erased when a new dot is plotted.

k is a toggle, so if you enter k again this feature turns off.

KK, KK1
When command kk is entered, a permanent cross turns on at the current position y (= y0) of the trajectory. "Permanent" means it is also drawn on the core copy of the picture.

The command kk1 draws the permanent cross at y1 instead of y (= y0).

Basic commands for drawing axes (in axes menu axm)

<table>
<thead>
<tr>
<th>Command</th>
<th>Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>XAX</td>
<td>X AXis</td>
</tr>
<tr>
<td>XAX1</td>
<td>X Axis with tic marks</td>
</tr>
<tr>
<td>XAX2</td>
<td>X Axis with double tic marks</td>
</tr>
</tbody>
</table>

XAX, XAX1, XAX2
The command xax causes the x axis to be drawn. The command xax1 draws the x axis and then draws tic marks, from 4 to 9 tic marks on a side, choosing the number so as to give natural divisions.

The command xax2 draws the x axis and then draws a double set of tic marks. The first set is identical with the tic marks of command xax1 and the second set is a finer set of small tic marks.

YAX, YAX1, YAX2
The commands yax, yax1 and yax2 are analogous to the x axis commands xax, xax1 and xax2.

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Basic commands for setting parameters (in parameter menu pm)

```
<table>
<thead>
<tr>
<th>Command</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>XCO</td>
<td>X COordinate to be plotted: y[0]</td>
</tr>
<tr>
<td>XS</td>
<td>X Scale: from -2.5 to 2.5</td>
</tr>
<tr>
<td>YCO</td>
<td>Y COordinate to be plotted: y[1]</td>
</tr>
<tr>
<td>YS</td>
<td>Y Scale: from -2.5 to 2.5</td>
</tr>
<tr>
<td>Cl</td>
<td>Cl = -0.3000000</td>
</tr>
<tr>
<td>RHO</td>
<td>rho = 2.1200000</td>
</tr>
</tbody>
</table>
```

The process parameters are set using c₁, c₂, ... , rho, etc. Only those process parameters that are used in the equations appear in the parameter menu. Enter the parameter's name (like rho) and the program will prompt for a new value.

Basic commands for setting parameters not being process parameters

**XS, YS**

The command xs allows you to change the horizontal scale of the screen for plotting dots; the first number is the left hand side and the second is the right one. The two numbers must be entered with a space but no comma between them. (The first number can be larger than the second one if you want a decreasing scale.)

The command ys allows you to change the vertical scale of the screen. The first number is the coordinate of the bottom of the screen and the second one is the top.

**XCO, YCO**

Most variables take on numerical values. The value of the variable xco is a name, the name of the horizontal x coordinate variable. This command is for specifying the variable or parameter that is to be plotted horizontally. For the Hénon map, the default value of xco is "y[0]", which is the first coordinate of the vector y. With this setting the numerical value of y[0] is plotted horizontally. The value of xco can either be a coordinate of the process (like y[0]) or a process parameter (like rho or c₁). To specify a coordinate like y[1] for the horizontal axis, type xco<Enter> and then enter just the number of the coordinate. In the case of y[1], type xco<Enter> 1<Enter> . To plot a parameter of the process, either rho, or c₁, c₂, ... c₉, type the process parameter and <Enter>. For example, xco<Enter> c₁<Enter> .

If you choose a parameter and plot a trajectory, then the trajectory will appear on a vertical line, the line corresponding to that parameter value.

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The value of yeo is a name, the name of the vertical y coordinate variable. This command is for setting the variable or parameter that is to be plotted vertically, and is otherwise similar to xeo. You can even assign one parameter horizontally (using xeo) and another vertically in order to plot analogues of Mandelbrot sets (using command bas or other related commands in the basin of attraction menu, see bm).

Both xeo and yeo can be set equal to the same variable. Suppose for example, both coordinates are set to be variable 0. Then the program plots vertically the specified coordinate of the trajectory, but horizontally it plots that coordinate from the previous time. That is, the vertical coordinate is delayed. To make the delay several time steps instead of 1, use ipp to change the number of iterates per plot.

See prn for the vertical coordinate in bifurcation diagrams.

Basic commands for changing variables (in when & what menu wwm)

<table>
<thead>
<tr>
<th>Command</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>CON</td>
<td>CONnect consecutive dots: OFF</td>
</tr>
<tr>
<td>PI</td>
<td>PreIerates before plotting: 0</td>
</tr>
<tr>
<td>PRM</td>
<td>PaRaMeter to be varied by +/-: &quot;rho&quot;</td>
</tr>
<tr>
<td>PT</td>
<td>T Plots Time horizontally if ON; now OFF</td>
</tr>
</tbody>
</table>

CON

The command con causes successive dots to be connected by straight lines. It can also be used for maps when it is appropriate.

con is a toggle: if con is on, enter it again and it turns off.

PI

The command pi is for setting the number of preiterates, that is, the number of iterates computed before the first point is plotted. When iterating a process, it may be iterated pi times before plotting begins. The number of preiterates pi should be at least 0. The usual default is 0.

PRM

The command prm is for specifying the parameter, for example, the parameter to be varied in bifurcation diagrams. The value of the variable prm is a name. The value of prm is the name of the parameter to be varied.

PT

The command pt is a toggle. When pt is on, the trajectory command t plots the time horizontally. Vertically it plots y[1] (assuming l = 0, that is, the Lyapunov exponents are not being computed). The consecutive dots can be connected by invoking the "connect consecutive dots" command con.
Basic commands for differential equations (in differential eqs. menu dem)

<table>
<thead>
<tr>
<th>Command</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>EULER</td>
<td>Euler solver, fixed step size</td>
</tr>
<tr>
<td>RK4</td>
<td>4th order Runge-Kutta solver, fixed step size</td>
</tr>
<tr>
<td>DF</td>
<td>plot the Direction Field</td>
</tr>
<tr>
<td>STEP</td>
<td>STEP size for differential equation: 0.01</td>
</tr>
</tbody>
</table>

**DF**

The command df is for plotting the direction field of a differential equation. See also command fg.

**EULER**

The command euler is for choosing Euler differential equation solver. This command makes the differential equation solver a single step Euler solver.

**FG**

The command fg is for specifying the grid used by df (direction field) and vf (vector field). This command is in the actions, hints and options menu for df.

**RK4**

This command makes the differential equation solver a fourth order fixed-step-size Runge-Kutta solver. This is the usual default solver.

**STEP**

The command step sets the step size of the differential equation solvers rk4 and euler. Depending on the differential equation(s) you are exploring, in order to get reliable numerical results, you may have to set step to a much smaller value; for example, you may have to set step to 0.0001 or 0.00001.
Commands for graphs, own, pause, quit and trajectories

DYN

The command dyn is used to quit and to restart Dynamics. This command allows selection of another differential equation or map; most parameters are reset to the default values.

G

The "plot graph" command g plots the graph of 1-dimensional functions. The routine g calculates the function value for a fixed number of points (see wide in the size of core menu to find this value).

If the iterates per plot ipp is changed from 1 to 3, for example, then g will graph the third iterate of the function. You may wish to connect the consecutive dots by first entering the command con. See command con in wwm. Enter con again after creating the graph to turn that feature off.

OWN

The command own is used to quit the program Dynamics for creating your own process (map or differential equation).

P

The command P causes the program to Pause. Hit the <space bar> or enter the "un-pause" command up to end pause mode. If you are going to start a process like t or um and you want it to be paused initially, then this is the command to use. The pause initiated by <·> while plotting has a different effect: the program will automatically un-pause from the <·> pause if you start a new process.

Q, QX

The command q is for quitting the program. Enter this command and a menu appears on the screen before exiting the program, to allow saving of the data.

The command qx is for quitting and exiting Dynamics. This command terminates the program; data that has not been saved will be lost.

T

The "trajectory" command instructs the computer to plot the dots of a trajectory. The maximum number is specified by the command dots. Usually dots is set to such a large value that it will not be reached. Use x coordinate and y coordinate commands xco and yco to specify which coordinates will be plotted against which horizontally and vertically. See xco. Usually xco and yco are set to be the space coordinates of the process. For example, in Henon the default setting of xco is 0, which means y[0] is plotted horizontally, and yco is 1 which means y[1] is plotted
vertically. However as shown below xco and/or yco can be set to be parameters of the map.

After entering the trajectory command t, an options menu appears. One of the options is to follow a number (tn) of trajectories simultaneously. Usually routine t plots a single trajectory, but if the trajectory number tn is greater than 1, the routine t plots tn trajectories simultaneously. If you set tn to be 100 for example, 100 initial points will be chosen. See commands tn and tnb for more information.

You can specify that one of the axes is a parameter of the system, for example the horizontal axis could be for the parameter rho (if that is one of the parameters of the currently selected process), and then rho would be varied over a range specified using the "x scale" command xs. Of course, the trajectory would then lie on a single line corresponding to that parameter value. The sequence of commands

\[ xco < Enter > \quad \text{rho} < Enter > \quad yco < Enter > \quad 1 < Enter > \]

says that xco is rho, that is, rho will be plotted horizontally, that is the x coordinate, while coordinate 1 of the state vector will be plotted vertically.

**T, TN, TNB**

The "trajectory number" command tn specifies the number of trajectories that the routine t will follow simultaneously. The "tn box" command tnb is a toggle; it is either "on" or "off".

If \( \text{tn} = 1 \), then the routine t plots a single trajectory starting from the point \( y[1] \). If \( \text{tn} > 1 \), then the routine t plots \( \text{tn} \) trajectories simultaneously; each time you enter command t, these \( \text{tn} \) initial points are chosen anew. The initial conditions can be either on the line from \( y_a \) to \( y_b \) if \( \text{tnb} \) is "off", or if \( \text{tnb} \) is "on", they are chosen in the "small box".

When \( \text{tn} > 1 \) and \( \text{tnb} \) is off, the trajectory command t chooses \( \text{tn} \) initial points equally spaced points are chosen on the segment from \( y_a \) to \( y_b \). These points include \( y_a \) and \( y_b \), and they are used as initial points for trajectories. If you set \( \text{tn} \) to be 100 for example, 100 initial points will be chosen on the line from \( y_a \) to \( y_b \). (See y vectors command yv to see what these values are at any time. Initially they are in the lower left and upper right corners of the screen so the line is the diagonal connecting them.) Then t will follow 100 trajectories simultaneously. You can see the process more clearly if before entering the command t you first enter the pause command p. Then when t is entered, the program will be paused. Hit the period key <.> and all 100 will be advanced one iterate. Hit <.> again and all will advance one more step.

When \( \text{tn} > 1 \) and \( \text{tnb} \) is on, the trajectory command t chooses \( \text{tn} \) points at random from a rectangle which is either the current window (or whole screen) or if the small box has been set, it chooses the initial points in that box. You can see the small box, if it has been set, using command b.
1.4 SOME BASIC INTERRUPTS

Recall that "Enter a command" means type the command (using upper or lower case) and then hit the <Enter> key. A few commands (called interrupts) are executed without hitting <Enter> and are called by a single key stroke such as <·>, <Tab>, <Esc>, <space bar>, and the arrow keys.

<·>

The key <·> (one dot) pauses the program Dynamics after plotting one dot. Each time <·> is hit, the program computes one more dot and plots it and pauses. The large cross appears at the current location of the trajectory. Holding <·> down will produce a string of iterates; <space bar> returns the program to normal continuous plotting of dots.

<space bar>

<space bar> ends the pause mode; see also command p (pause) and interrupts <·> and <Enter> which each initiate a pause mode.

<Esc>

When <Esc> is hit, the current routine terminates or if a menu is on the screen, it terminates the menu and retrieves the parent menu. Hit it again and the program calls its parent menu.

<Tab>

If a process is being run and you hit <Tab>, the program prints the speed (in dots per second) and a selection of parameter values.

If a menu is on the screen, hitting <Tab> will fetch information about the high-lighted command with information.

<F1>, <F2>, <F3>, <F4>, <F10>

The Function keys <F1>, <F2>, <F3>, and <F4> can be used to switch back and forth between windows that have been opened. Use <F10> to switch back to the whole screen.

Each window is a quadrant of the screen, situated as follows:

<F1> <F2>  
<F3> <F4>

<F7>, <F8>, <F9>

The color numbers on the screen are usually numbered 0 to 15. The "color table" command ct displays the color table. The program has an active color number that is used in all plotting (and in refreshing the screen). Function key interrupt <F9> is used for setting the value and the program will prompt the user for a color number; <F8> increases that number while <F7> decreases it.
1.5 USING THE MOUSE
The mouse can be used for a variety of purposes. It is not essential for Dynamics, but it can simplify procedures.

Selecting items in menus (left mouse button)
If there is a menu on the screen, you can select an item in that menu using the left mouse button. If you click on a command that is not illuminated, it will become illuminated. If it is illuminated and you click on it, then that command will be executed.

Picking a new initial point of a trajectory (left mouse button)
If no menu is displayed on the screen, then the mouse can be used to set the initializer y1. (If a menu is displayed and you want it to vanish, just hit the <space bar>). Move the mouse and the mouse arrow will appear. Move it to the position on the screen that you select to be the new initial point. When you have selected a new initial point, double click the left mouse button. If you are plotting a trajectory, this will reinitialize the trajectory and plotting will continue from this newly selected point.

If you are not using a mouse, the same can be accomplished by using the arrow keys to move the small cross, and then entering command i (initialize).