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RECURRENT OUTBREAKS OF MEASLES, CHICKENPOX AND MUMPS

II. SYSTEMATIC DIFFERENCES IN CONTACT RATES AND STOCHASTIC EFFECTS[1]

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Yorke, J. A. and W. P. London (Mathematical Research Branch, National Institute of Arthritis, Metabolism, and Digestive Diseases, Bethesda, Maryland 20014). Recurrent outbreaks of measles, chickenpox and mumps. II. Systematic differences in contact rates and stochastic effects. *Am J Epidemiol* 98:469–482, 1973.—The mean monthly contact rates for measles, chickenpox and mumps estimated from the monthly reported cases show systematic differences between the years with many cases and the years with few cases. In New York City, the mean contact rates for chickenpox were different during the years 1931–1945 than during 1946–1960. The clustering of cases within social groups is proposed to account for these differences in the contact rates and for other empirical observations. The irregularity of outbreaks of measles in cities of fewer than two million people can be explained by stochastic effects. Outbreaks of measles in distant large metropolitan areas are highly correlated in time, but the reasons for the correlation are not clear.

chickenpox; communicable disease; disease outbreaks; epidemiologic methods; measles; models, theoretical; mumps; varicella

INTRODUCTION

In the previous paper (1) mean monthly contact rates were estimated from reported monthly cases of measles, chickenpox and mumps and were shown to be substantially higher in the autumn and winter months. In this paper we discuss systematic differences that occur in the mean monthly contact rates between the years with many cases vs. the years with fewer cases, and for chickenpox between the early and later years of study. In the section on "Nonhomogeneous populations" it is shown that subpopulations with different degrees of sociability or susceptibility or the grouping of children in grades in school do not appear to account for the systematic differences in the contact rates. The clustering of cases within social groups of close acquaintances in, for example, households, playgroups or classrooms is proposed to account for the systematic differences and for other empirical observations. The clustering effect is found to be greatest for chickenpox and least for the most infectious disease, measles.

Stochastic effects are shown below to be consistent with the irregularity of outbreak

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of measles in cities of fewer than two million people. Data are presented that show that outbreaks of measles in distant large metropolitan areas are correlated in time. The reported monthly cases of measles, chickenpox and mumps in New York City and measles in Baltimore from 1928 to 1972 are presented in Appendix 1.

SYSTEMATIC DIFFERENCES IN THE MEAN MONTHLY CONTACT RATES

The technique of estimating mean monthly contact rates from 30 or 35 years of data of the reported monthly cases of each disease is discussed in the previous paper (1). As before, the disease year is defined as the 12 months from September 1 through August 31. A high year is a disease year in which many cases were reported; a low year is a disease year in which relatively few cases were reported. The number of reported cases that distinguish high years from low years is given in the legend to figure 1. A contact or exposure is defined as a successful transmission of the infection.

Systematic differences between the high and low years. The monthly contact rates for measles, chickenpox and mumps show systematic differences between the years with many cases vs. the years with fewer cases (figure 1). During the months November, December and January in New York City the contact rate for measles for the high years is systematically and significantly higher ($p < .05$) than the contact rate for the low years. (The levels of statistical significance correspond to the month or months with the maximum t value. Since the average contact rates from successive months show strong positive dependence, there is partial justification for the use of the maximum t.) The same is true for the contact rate for Baltimore in October, November and December. In New York

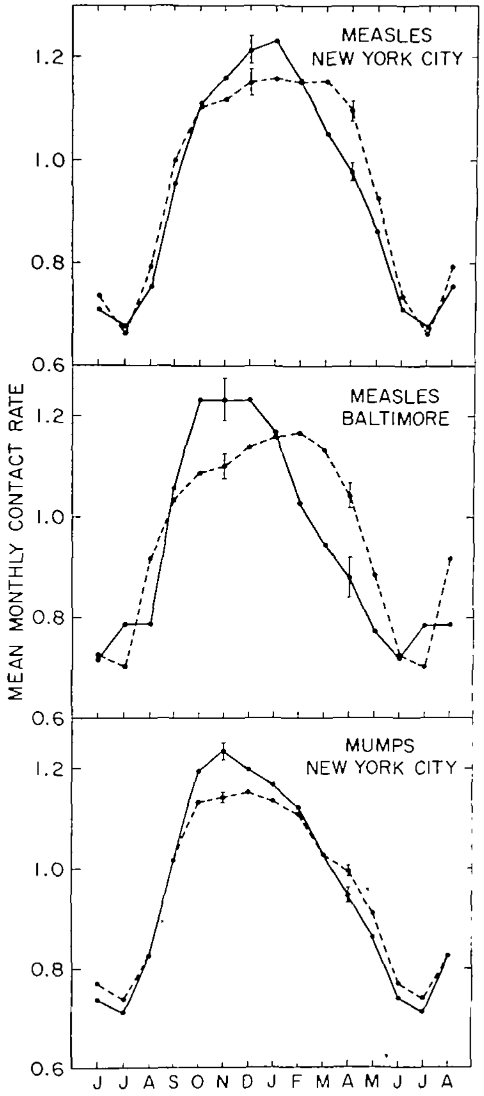

FIGURE 1. Mean monthly contact rates from the high and low years. The solid line is the contact rate from the high years, the dashed line is the rate from the low years. For each disease the contact rates are normalized by the average of the 12 mean monthly contact rates from all years (figure 2 in reference 1). The bars show one standard error on either side of the mean. Because of delays due to the incubation period and in reporting, the notifications from one month correspond to the contact rate of the previous month. Further details are given in the legend to figure 2 in reference 1. The range of the annual number of reported cases are: measles in New York City, high years: 21,000–42,000; low years: 1,500–13,000; measles in Baltimore, high years: 4,900–19,000, low years: 86–3,800; mumps in New York City, high years: 6,100–12,000, low years: 2,700–5,600; chickenpox in New York City (not shown, but discussed in the text) high years: 9,700–13,000, low years: 3,300–9,300.

City during these months the maximum differences between the two mean monthly contact rates is 6 per cent, in Baltimore, 10 per cent. In March, April and May in New York City (in Baltimore, February through May) the monthly contact rate from the high years is systematically and significantly lower ($p < .005$) than the monthly contact rates from the low years. The maximum difference between the two monthly contact rates is 11 per cent in New York City, in Baltimore, 16 per cent. In New York City the monthly contact rate for measles from the low years is essentially constant from October to April. For mumps the monthly contact rates from the 17 high years are 6–8 per cent higher during October and November ($p < .001$) and 4–5 per cent lower in April and May ($p < .025$) than the contact rates from the 13 low years. For chickenpox (not shown) the monthly contact rates from the 14 high years are about 5 per cent higher in September and October ($p < .01$) and about 4 per cent lower in May and June ($p < .05$) than the mean monthly contact rates from the 16 low years.

The enhanced contact rate in the early phase of the outbreak in the high years and the diminished contact rate in the late phase of the outbreak in the high years are similar to the findings of Abbey (2), who, using the Reed-Frost model, studied single outbreaks in populations fewer than 1000 people in schools or institutions. The fit of those data to that model was improved by assuming (contrary to fact) that initially the total population was susceptible and that at the end of the outbreak no susceptibles remained. In our studies, increasing the number of susceptibles in the calculation of the monthly contact rates did not remove the difference between the high and low years, and decreasing the number of susceptibles (to enhance depletion of susceptibles) raised the contact rates in June, July and August but not significantly during March, April and May. Likewise, if the incubation period of measles was decreased from 12–13 days to 8–10 days (to allow the outbreak to spread and die out more rapidly) the systematic differences in the autumn months were decreased but not eliminated and the systematic differences in the spring remained. Contact rates calculated with an incubation period longer than 12–13 days, or with a distribution of incubation periods (see reference 1), or with the models of ordinary differential equations that assume an incubation period (see appendix of reference 1) show the same systematic differences as the curves in figure 1. The systematic differences in the contact rates between the high and low years might be an anomaly of the estimation procedure; calculation of contact rates using monthly totals generated by the stochastic version of the model (see the section on "Stochastic effects") suggests, however, that the systematic differences are not artifactual. Finally, it is unlikely that systematic differences in the fraction of cases reported during the course of the outbreaks can account for the systematic differences in the contact rates.

Modified exposure rates. The systematic differences in the measles contact rate between the high and low years (figure 1) were nearly eliminated by arbitrarily modifying the exposure rate formula (equation 1 in reference 1) to be $\beta(t)S(t)I(t)[1 + cI(t)\ (St) - S_o)/S_o]$, where c and S_o are constants chosen to minimize the systematic differences. The maximum modification of the exposure rate is 10 per cent. In the autumn months before the peak of the outbreak $S(t)$ exceeds S_o and the positive modification lowers the contact rate in the high years. After the peak of the outbreak in the spring months S_o exceeds $S(t)$ and the negative modification raises the contact rate in the high year. The parameter c can be chosen so that the number of infectives in the low years is too small for the modification to be significant. A simpler modified exposure rate, $\beta(t)S(t)I(t)\ (1 - cI(t))$, nearly eliminates the systematic differences in the spring months but the differences in

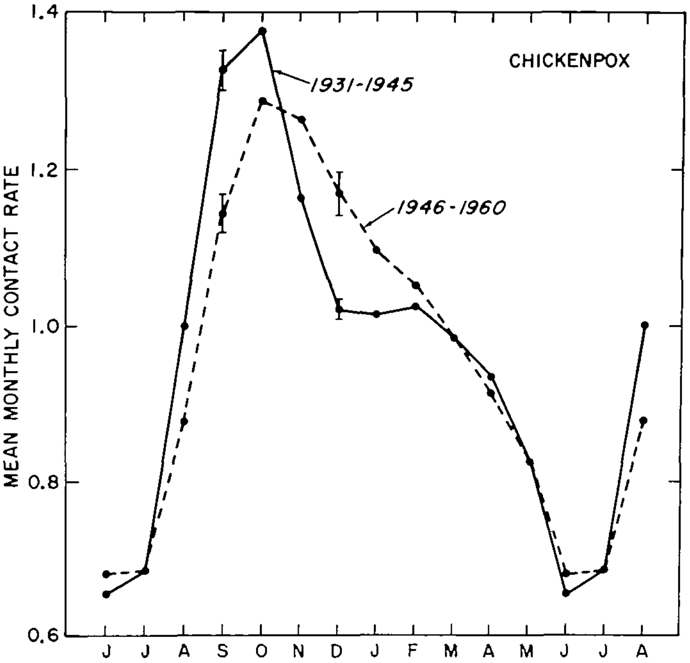

FIGURE 2. Mean monthly contact rates for chickenpox in New York City: the early years vs. the late years. The bars show one standard error on either side of the mean. The contact rates are normalized by the average of the 12 mean monthly contact rates from the 30 years (6.07×10^{-6}). Because of delays due to the incubation period and in the reporting, the notifications from one month correspond to the contact rate of the previous month. Further details are given in the legend to figure 2 in reference 1.

the autumn months are slightly increased. Although these modifications are effective in eliminating the systematic differences in the mean monthly contact rates we are not able to interpret the modifications on biologic or physical grounds.

Systematic differences between the early and later years. The monthly contact rates for chickenpox in New York City show systematic differences between the 15 early years (1931–1945) and the 15 later years (1946–1960) (figure 2). In the early years the mean monthly contact rate rises to a higher peak in October and then falls below the contact rate from the later years in months November through February. The maximum difference between the two curves is about 15 per cent. (From August through January all differences are statistically significant ($p < .025$).) Similar differences between the early and the later years are seen in the mean monthly contact rates calculated for the three largest boroughs: the contact rates from Manhattan show the largest differences, those from Bronx, the smallest differences. (Data from Queens and Richmond, the boroughs with the least number of reported cases, were not analyzed.) The mean monthly contact rates for measles (both in New York City and Baltimore) and for mumps do not show the systematic differences between the early and late years of study.

NONHOMOGENEOUS POPULATIONS

The delay equation model of the previous paper (1) that is based on random mixing of susceptibles and infectives in a single homogeneous population yields satisfactory curves of mean monthly contact rates and satisfactory simulations of the recurrent outbreaks. The systematic differences in the contact rates between the high and the low years (figure 1), differences of at most 16 per cent, suggest, however, that the simple model is not adequate. We now consider possibly more realistic models that include subpopulations, or the grouping of children in school, or the clustering of cases within social groups.

Subpopulations. One possible explanation for the systematic differences in the mean monthly contacts between the high and low years is based on subgroups or subpopulations with different susceptibility to the disease or different social characteristics. We assume one group of children who are more social or more susceptible than average (group I) and another group of the remaining children who are less social or less susceptible than average (group II). Based on these two groups a single contact rate for the total population was calculated for each month by increasing the number of equations in the model and specifying time independent intragroup and intergroup coefficients that modeled increased or decreased susceptibility or sociability. At the beginning of each disease year susceptibles were added to each group. As expected, early in a high year the majority of exposures were observed in group I and late in a high year most exposures occurred in group II, but the systematic differences in the mean monthly contacts were not eliminated: the differences in the autumn months were reduced only slightly and differences in the spring months remained.

The grouping of children into grades in a school was also modeled by having eight or 10 subpopulations. At the end of each disease year, the susceptibles wére "promoted" into the next higher grade; those in the highest grade were eliminated, and new susceptibles were introduced into the lowest grade. Various degrees of contact among the grades were tried but regardless of the coupling, the mean monthly contact rates showed the original systematic differences between the high and low years.

We conclude that the systematic differences in the mean monthly contact rates between the high and low years cannot be accounted for by two subpopulations with different degrees of sociability or susceptibility or by the grouping of children in grades in school.

Clustering of cases and the role of the casual acquaintance. The clustering of cases within social groups is another possible explanation for the systematic differences in the contact rates and for other observations. Two individuals are said to be close acquaintances if they belong to the same household, playgroup, classroom, neighborhood or community; two individuals are casual acquaintances if they are not close acquaintances. Over a period of weeks or months cases will cluster nonuniformly in one set of close acquaintances while other virtually identical subgroups will remain relatively free of the disease. Similar notions of clustering are discussed by Elveback et al. (3).

A critical factor in determining whether cases will cluster is the ratio F of casual acquaintances to close acquaintances who are contacted by an average infective during his period of infectivity. Individuals contacted more than once are counted only once; F is independent of the ratios of susceptibles to immunes in the group of close acquaintances and in the entire population. The more infectious diseases have larger ratios F, because more casual acquaintances are contacted. Indeed, an increased number of different casual acquaintances are contacted, while multiple contacts of close acquaintances produce little increase in the number infected.

Table 1, which compares the infectivity of measles, chickenpox and mumps in society and in the household, shows that clustering is more important for the less infectious diseases. Here the group of close acquaintances is the household. Both in society and in the household measles is the most contagious and mumps the least contagious of the three diseases. In comparison with measles, chickenpox is about 80 per

TABLE 1

Infectivity of measles, chickenpox and mumps

Disease	Exposures/susceptibles					
	In society				In households	
		%†		%†		%†
Measles	0.26*	100	0.26‡	100	0.76§	100
Chickenpox	0.12 (0.09, 0.17)	46 (35, 65)	0.17	65	0.61	80
Mumps	0.075 (0.05, 0.11)	29 (19, 42)	0.066	25	0.32	42

* Annual exposures divided by average number of susceptibles; calculated from the differential delay equation model 1 as $\bar{\beta} T_2 \gamma$ where $\bar{\beta}$ is the average of the 12 mean monthly contact rates, T_2 the period of infectivity, and γ the net input of susceptibles. If the contact rate is constant the reciprocal of the ratio of exposures to susceptibles is the mean number of years a susceptible is active in the population before contracting each disease. The calculation of the average number of susceptibles $(1/(\bar{\beta} T_2)$ depends on the assumed number of susceptibles at the peak S_p, which for chickenpox and mumps is difficult to know. For these diseases the ratios in parentheses are calculated using values of S_p one-third larger and one-third smaller than the choice of S_p in the previous paper (1).

† The ratio of exposures to susceptibles for each disease divided by that for measles times 100.

‡ Calculated from age specific attack rates for children aged 9–15 (1935–1936, 28 urban areas) (4).

§ The fraction of contacts of susceptibles in an infective's household that lead to transmission of the disease (5).

cent as infectious in the household but only 35 to 65 per cent as infectious in society. Within the household, only close acquaintances are counted and chickenpox appears relatively more contagious; in society, where both close and casual acquaintances are counted, chickenpox appears relatively less contagious because the disease does not spread readily among casual acquaintances. A similar argument can be made for mumps, which in comparison with measles, is relatively more contagious in the household than in society. A similar pattern should occur for other social groups such as playgroups, classrooms and neighborhoods.

The clustering of cases is consistent with other observations:

1) As shown in figure 3, the outbreaks of measles spread simultaneously through the four major boroughs in New York City. The peaks of the outbreaks in each borough occur within one month of each other, and the high years for the boroughs coincide. In contrast, the outbreaks of chickenpox and mumps spread less uniformly through the boroughs. The tendency for contacts of measles, the most contagious of the three infections, to be made more often among casual acquaintances outside the cluster of close acquaintances strongly increases the uniformity of the spread of the infection.

2) The clustering of cases is also consistent with the decrease in the contact rate in the spring months of the high years (figure 1). During these months after the disease has spread easily among and eventually depleted the susceptibles in the cluster of close acquaintances, the infectives are isolated among the immunes, while virtually identical clusters continue with many susceptibles; the contact rate calculated for the entire population is decreased. In a low year, too few cases occur to produce significant clustering of cases. The decline in the contact rate in the spring months in New York City is about 10 per cent for measles and about 5 per cent for chickenpox and mumps, but the ratio of cases in the average high year vs. the average low year is about 5:1 for measles but less than 2:1 for chickenpox and mumps. Thus, the clustering effect, as measured by the decline in the contact rate, is more impressive for chickenpox and mumps than for the most infectious disease, measles. Clustering does not, however, appear to explain the enhanced

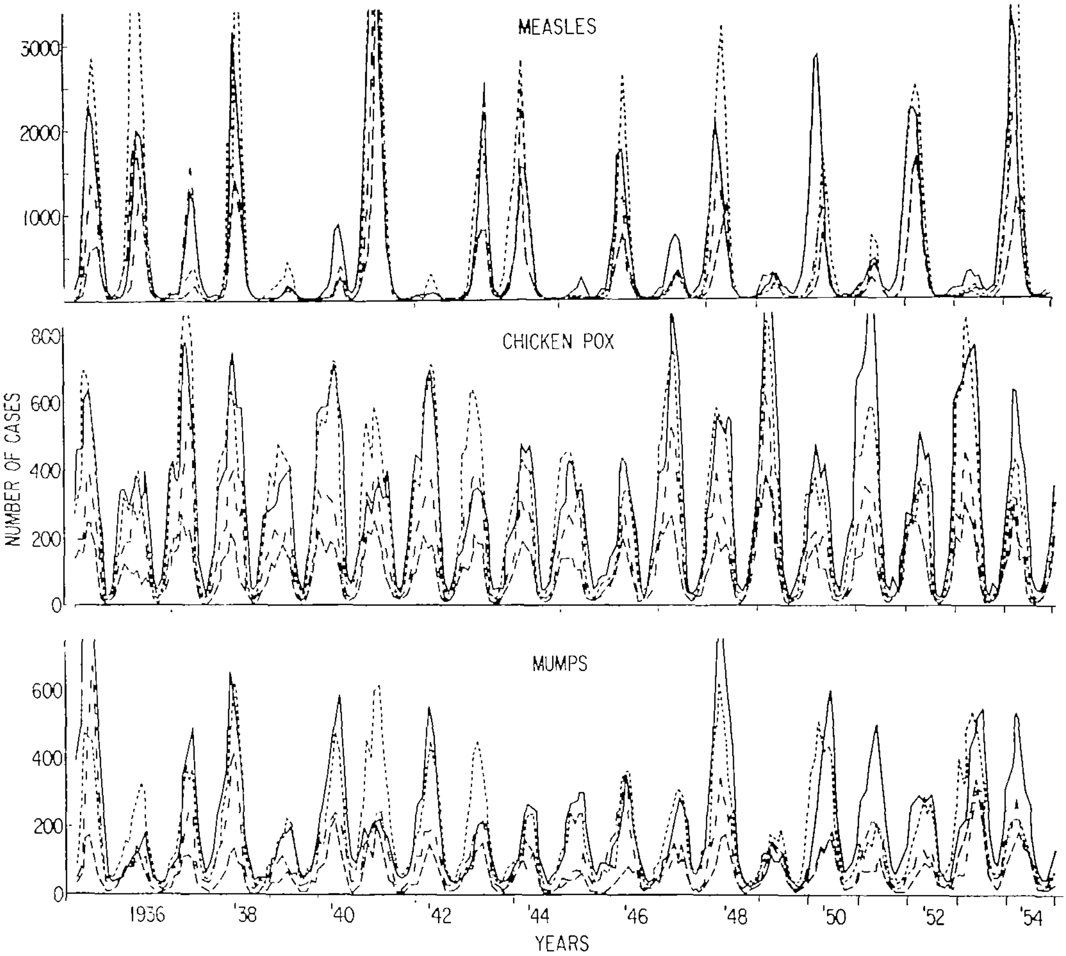

Figure 3. *Recurrent outbreaks of measles, chickenpox and mumps in the four largest* boroughs in New York City (1935–1954). Manhattan, ——; Brooklyn, ——; Bronx, -···-; and Queens, – – –.

contact rates in the autumn months of the high years. (We did not find it technically feasible to calculate contact rates with a model that included clustering, and the hypothesis was not tested formally.)

3) The clustering of cases might also explain the shape of the curve of the monthly contact rate for chickenpox and differences in the contact rates between the early and late years (figure 2). We speculate that in the early years (1931–1945) the sharp rise in the contact rate to a very high peak in October corresponds to the spread of infection within densely populated, closely knit, isolated neighborhoods or community clusters. The precipitous fall from October to December and the low level that is maintained until March or April correspond to the depletion of susceptibles within the cluster and the failure of the infectives to contact susceptibles in other clusters. The failure of the outbreak to spread in spite of adequate numbers of infectives and susceptibles is reflected in the flattened or truncated waves of outbreaks of chickenpox in figure 3. The individual boroughs in New York City show the same truncated waves and the same decrease in the contact rate from October to December, and so the proposed clustering effect is

within each borough. During the later years (1946–1960) we speculate that the neighborhood clusters were not so well defined and throughout the course of the outbreak the infection was spread among both close and casual acquaintances. One would not expect the contact rate for measles, the most contagious disease, to manifest these effects of clustering, but it is not clear why the effects are not seen in the contact rate for mumps, which is presumably the least contagious disease.

STOCHASTIC EFFECTS

In this section we consider a version of the delay equation model that includes stochastic effects. As in the previous paper (1) the difference delay equations are

$$E(t_n) = \beta(t_n)I(t_n)S(t_n)$$

$$S(t_{n+1}) = S(t_n) - E(t_n) + \gamma$$

$$I(t_{n+1}) = \sum_{i=T_1}^{T_1+T_2-1} E(t_{n+1-i})$$

where the fixed time interval or stepsize Δ equals one day, the nth time interval is t_n, E, S, I are the number of exposures, susceptibles and infectives, respectively, γ is the net input of susceptibles, β is the contact rate and T_1 and T_2 are the periods of incubation and infectivity, respectively.

If the expected number of exposures is $E_e(t_n)\Delta$, where Δ is the stepsize, the actual number of exposures, $E_r(t_n)\Delta$, should be modeled as a random number chosen from a binomial distribution (2). Since the probability of a susceptible being exposed during a Δ time period is very small (in the simulations $< 10^{-3}$ for $\Delta \leq 1$) the binomial distribution can be approximated by a Poisson distribution with mean $E_e(t_n)\Delta$. In the stochastic calculations (with $\Delta = 1$) we choose $E(t_n)$ randomly from a uniform distribution (to simplify calculations) with mean $\beta \cdot IS$ and standard deviation $(\beta \cdot IS)^{1/2}$. That is, let

$$E_e(t_n) = \beta(t_n)I(t_n)S(t_n)$$

and let R_n be a random number from a uniform distribution in the interval $(-3^{1/2}, +3^{1/2})$ so that the mean is zero and standard deviation 1. We set

$$E_r(t_n) = E_e(t_n) + R_n[E_e(t_n)]^{1/2}$$

We would expect that the stochastic perturbation decreases with the size of the population, and in fact, as shown by Bartlett (6), the larger the population, the less likely stochastic effects will terminate an outbreak. To model a population of M million people stochastically we let $E_e{}^*$ and $E_r{}^*$ be the expected and actual exposure rates per million people, respectively. For M million people the total exposure rates are $ME_e{}^*$ and $ME_r{}^*$. For R_n defined as before

$$ME_r{}^*(t_n) = ME_e{}^*(t_n) + R_n[ME_e{}^*(t_n)]^{1/2}$$

or

$$E_r{}^*(t_n) = E_e{}^*(t_n) + R_n[E_e{}^*(t_n)]^{1/2}M^{-1/2} \quad (1)$$

If $\beta(t)$ has been determined for a population of one million people, we simulate stochastically a population of M millions by equation 1 and

$$E_e{}^*(t_n) = \beta(t_n)I(t_n)S(t_n)$$

$$S(t_{n+1}) = S(t_n) - E_r{}^*(t_n) + \gamma$$

$$I(t_{n+1}) = \sum_{i=T_1}^{T_1+T_2-1} E_r{}^*(t_{n+1-i})$$

Periodicity of outbreaks and city size. Stochastic perturbations of the type studied here describe the recurrent outbreaks of measles similar to those of Baltimore that occurred at irregular intervals of one, two or three years. Simulations with stochastic effects that correspond to a population of 1.5–3 million, which is the approximate size of the Baltimore metropolitan area, yield outbreaks at irregular intervals. A typical string of 20 high (H) and low (L) years is

H-L-H-L-H-L-L-H-L-L-
H-L-H-L-H-L-H-H-L-H

Smaller stochastic perturbations, corresponding to a population greater than three million, yield less irregularity and, mainly, biennial outbreaks with an occasional extra high year are observed. Larger perturbations, corresponding to a population smaller than one million, yield more irregularity and after an exceptionally high year in the simulation the disease completely disappears.

In relating the periodicity of recurrent outbreaks of measles to the size of the city, New York City and Baltimore obviously exceed the critical size of 250,000 to 300,000 that assures the lack of "fade out" of outbreaks (defined as a one-month gap in the monthly notifications) (7). Bartlett (6) has also described a correlation between the periodicity of measles outbreaks and populations in the range of 1,000 to 500,000. The stochastic perturbations studied here show that mainly biennial outbreaks of measles should occur in metropolitan areas larger than 4 million people; in metropolitan areas of 2 to 4 million people (such as Baltimore) biennial and triennial outbreaks would be expected.

Assessment of the calculation of mean monthly contact rates. The stochastic version of the model was used in conjunction with the previously described method of calculation of mean monthly contact rates. A mean monthly contact rate for measles similar to that in figure 2 in the previous paper (1) was used to generate recurrent outbreaks with the stochastic version of the model. Mean monthly contact rates were then calculated from the resulting monthly totals of exposures by the usual technique. The original curve of the monthly contact rates that was used to generate the recurrent outbreaks was recovered in every month and there were no systematic differences between the high and low years. Likewise, a constant monthly contact rate was used to generate recurrent outbreaks, and again the constant contact rate without differences between high and low years was recovered in every month. These results suggest that stochastic effects of the type studied here are not responsible for the seasonal variation in the contact rates or for the differences in the contact rates and that the systematic differences in the contact rates are not an artifact of the method of estimating contact rates from the data of monthly notifications.

TABLE 2

Outbreaks of measles in four cities

City	1925	1926	1927	1928	1929	1930	1931	1932	1933	1934	1935	1936	1937	1938	1939
New York City		P*		P			P		P			P		P	
Philadelphia		P		P			P			P		P		P	
Baltimore		P		P			P			P			P		P
Washington, D.C.		P		P			P			P		P	#		P
	1940	1941	1942	1943	1944	1945	1946	1947	1948	1949	1950	1951	1952	1953	1954
New York City		P		#	P		P		P		P		P		P
Philadelphia		P		P			P		P		P		P		P
Baltimore			P		P		P			P			P		P
Washington, D.C.		P			P		P		P				P		P
	1955	1956	1957	1958	1959	1960	1961	1962	1963	1964	1965	1966	1967	1968	1969
New York City		P		P		P		P		P		P			P
Philadelphia		P		P			P	#		P		P			P
Baltimore		P		P		P	#			P		P			
Washington, D.C.		P		P		P		P		P		P			

* P = a peak year in which the number of reported cases exceeded the preceding and succeeding year; # = a year in which the number of reported cases was at least 90 per cent of the preceding or succeeding year.

THE COUPLING OF OUTBREAKS IN LARGE METROPOLITAN AREAS

Outbreaks of measles in one area may be influenced by neighboring metropolitan areas. Bartlett (6), for example, mentions the linkage of small towns in Wales; several cities in the United States regularly had coincident outbreaks of measles in the 1920's (8). Table 2 shows that since 1925 outbreaks of measles in New York City, Philadelphia, Baltimore and Washington, D.C. coincide surprisingly often. The correlation coefficients (table 3) are all positive. Since successive peaks in each city are not independently distributed, it is difficult to calculate the chance that these coincidences are so frequent; in any case the probability that New York City correlates positively with the other three cities is 0.125. The correlations are not related to the distances between the cities (the four cities lie on a line, the distances from New York to Philadelphia, Baltimore and Washington being about 161, 322.0 and 386.40 km, respectively). The coupling of these cities could be due to travel by infectives, the simultaneous migration of susceptibles into or from these cities or common weather conditions, but none of these effects seems strong enough to account for the observed correlations. No pattern of recurrent biennial outbreaks of measles is evident in the statistics for the entire country (9).

TABLE 3

Correlation of the outbreaks of measles in four cities

City	No. of high years*	No. of high years in common (correlation coefficient)		
		New York City	Philadelphia	Baltimore
New York City	21			
Philadelphia	20	18 (0.78)		
Baltimore	18	12 (0.33)	12 (0.37)	
Washington, D.C.	19	16 (0.64)	15 (0.59)	15 (0.68)

* High years are the years in table 2 indicated by "P" or "$\frac{P}{2}$." Correlations were made on the sequences of 45 ones and zeroes obtained for each city, where one indicates a high year and zero, a low year. Similar results are obtained if only the peak years in table 2 are used as high years.

REFERENCES

1. London WP, Yorke JA: Recurrent outbreaks of measles, chickenpox and mumps. I. Seasonal variation in contact rates. Am J Epidemiol 98: 453–468, 1973
2. Abbey H: An examination of the Reed-Frost theory of epidemics. Hum Biol 24:201–233, 1952
3. Elveback L, Ackerman E, Gatewood L, et al: Stochastic two-agent epidemic simulation models for a community of families. Am J Epidemiol 93:267–280, 1971
4. Collins SD, Wheeler RE, Shannon RD: The occurrence of whooping cough, chickenpox, mumps, measles and German measles in 200,000 surveyed families in 28 large cities. Special Study Series, No 1, Division of Public Health Methods, NIH, Washington, USPHS, 1942
5. Hope Simpson RE: Infectiousness of communicable diseases in the household (measles, chickenpox, and mumps). Lancet 2:549–554, 1952
6. Bartlett MS: The critical community size for measles in the United States. J R Stat Soc Ser A 123:37–44, 1960
7. Bartlett MS: Measles periodicity and community size. J R Stat Soc Ser A 120:48–70, 1957
8. Emerson H: Measles and whooping cough. Am J Public Health (Suppl) 27:1–153, 1937
9. Center for Disease Control. Annual Supplements to Morbidity and Mortality Weekly Reports (1943–1963)

APPENDIX 1

Reported monthly cases of measles, mumps and chickenpox in New York City and measles in Baltimore

Measles—New York City

Year	Jan	Feb	Mar	Apr	May	June	July	Aug	Sept	Oct	Nov	Dec	Total
1928	609	1,516	4,952	7,466	11,155	7,002	1,315	189	74	119	287	324	35,008
1929	288	241	347	378	498	324	151	69	44	42	34	121	2,537
1930	316	781	1,870	5,309	7,499	5,386	1,386	211	107	128	219	423	23,635
1931	750	2,010	4,858	6,172	7,095	4,238	907	165	43	54	93	134	26,519
1932	146	270	621	1,096	2,271	2,537	1,081	288	118	149	594	1,183	10,354
1933	1,960	4,699	9,635	9,573	6,605	2,697	601	139	50	55	71	131	36,216
1934	133	168	431	652	1,414	1,317	422	95	47	45	86	205	5,015
1935	508	1,576	4,357	6,630	6,813	5,182	1,975	358	67	136	333	555	28,490
1936	1,426	4,054	7,376	8,792	8,247	4,738	1,143	213	87	99	154	219	36,548
1937	235	365	1,075	2,253	3,546	3,058	880	247	84	90	136	122	12,091
1938	519	1,585	5,713	10,018	8,634	6,253	1,231	239	75	56	110	171	34,604
1939	224	219	361	517	969	738	274	110	36	54	60	91	3,653
1940	90	207	268	533	1,365	1,906	1,269	307	151	354	1,050	2,996	10,946
1941*	6,336	13,226	25,826	22,741	8,195	2,527	436	100	63	67	55	74	79,646
1942	137	169	233	404	557	387	127	60	34	45	49	59	2,261
1943	232	877	2,174	3,855	5,724	5,760	1,759	453	142	300	676	1,675	23,627
1944	2,567	5,242	8,498	5,458	2,779	1,151	182	63	21	26	30	31	26,048
1945	60	82	171	207	221	358	194	68	18	79	181	231	1,870
1946	809	2,022	5,014	6,597	4,759	2,191	596	107	68	56	75	114	22,408
1947	254	379	669	1,141	1,682	1,613	854	202	88	174	234	625	7,915
1948	1,709	3,319	6,807	6,909	5,481	3,548	802	159	64	32	47	83	28,960
1949	169	390	629	693	1,008	960	454	234	133	100	214	483	5,467
1950	1,055	1,930	3,776	4,777	5,428	3,440	1,012	232	73	89	90	88	21,990
1951	282	381	549	1,079	1,915	1,770	948	380	184	246	515	1,360	9,609
1952	3,707	6,441	8,616	7,754	4,945	2,288	478	75	40	68	57	146	34,615
1953	224	246	334	518	645	600	522	215	164	209	318	1,122	5,117
1954	2,443	4,876	10,154	10,720	6,136	4,699	1,335	249	63	59	115	188	41,037
1955	364	603	1,305	1,853	1,865	1,797	760	245	103	98	242	477	9,712
1956	594	1,242	2,461	4,336	6,064	4,247	1,419	441	110	169	188	375	21,646
1957	823	789	949	1,287	1,949	1,840	845	335	176	170	231	832	10,226
1958	1,878	3,544	6,410	7,634	4,832	2,594	893	204	86	58	47	92	28,272
1959	199	323	537	683	716	753	567	174	144	97	187	837	5,217
1960	2,070	4,097	6,780	6,492	3,387	1,822	469	129	51	43	78	105	25,523
1961	227	298	374	384	644	683	343	185	109	123	383	1,043	4,796
1962	1,725	3,056	5,839	7,875	6,555	2,866	1,075	266	58	86	125	145	29,671
1963	184	260	476	782	1,200	1,289	901	362	168	221	423	1,140	7,406
1964	2,053	2,267	2,859	3,338	2,578	1,486	427	120	49	59	50	149	15,435
1965	120	126	150	354	419	656	331	174	83	120	395	1,199	4,127
1966	1,635	1,965	2,349	1,253	655	244	109	59	24	34	30	24	8,381
1967	39	52	57	78	83	79	35	28	18	11	27	22	529
1968	39	70	146	292	494	441	347	225	97	91	98	105	2,445
1969	278	384	768	1,301	1,019	788	267	76	31	39	36	46	5,033
1970	70	106	133	185	154	109	65	39	40	52	58	143	1,154
1971	283	557	818	844	501	472	206	60	23	22	12	21	3,819
1972	28	35	40	31	41	32							

Chickenpox—New York City

Year	Jan	Feb	Mar	Apr	May	June	July	Aug	Sept	Oct	Nov	Dec	Total
1928	757	873	1,049	706	716	558	201	45	39	293	682	1,163	7,082
1929	1,174	954	1,465	1,325	1,351	1,038	214	66	43	193	466	975	9,264
1930	1,034	1,002	1,179	1,129	927	777	168	46	66	152	560	861	7,901

Chickenpox—New York City

Year	Jan	Feb	Mar	Apr	May	June	July	Aug	Sept	Oct	Nov	Dec	Total
1931	956	927	1,585	1,536	1,448	1,272	303	68	62	116	275	565	9,113
1932	922	928	925	1,121	1,282	1,142	411	114	82	220	646	1,069	8,862
1933	1,320	1,473	2,305	2,094	1,694	1,043	390	127	56	148	426	890	11,966
1934	1,500	1,442	1,799	1,556	1,926	1,635	379	90	68	210	667	905	12,177
1935	1,124	1,192	1,850	1,941	1,505	1,016	429	58	78	251	605	817	10,866
1936	970	860	977	1,143	920	940	426	193	99	186	525	1,085	8,324
1937	1,220	1,157	1,974	2,301	2,277	1,746	413	129	78	160	448	820	12,723
1938	1,154	1,277	1,841	1,981	1,304	1,288	387	114	97	278	604	787	11,112
1939	1,010	968	1,195	1,200	1,218	1,183	334	145	66	252	536	996	9,103
1940	1,624	1,626	1,603	1,740	1,900	1,424	711	191	135	302	612	1,178	13,046
1941	1,409	1,218	1,543	1,477	987	935	495	126	109	197	397	880	9,773
1942	1,097	1,164	1,652	1,800	1,941	1,419	444	136	70	171	424	660	10,978
1943	1,050	1,177	1,559	1,513	1,371	1,042	205	67	83	143	469	611	9,290
1944	745	1,039	1,389	1,284	1,288	871	299	87	89	155	446	749	8,441
1945	1,037	1,080	1,289	1,211	1,076	1,080	372	132	78	133	203	214	7,905
1946	347	407	780	1,182	1,082	899	479	123	86	180	326	695	6,586
1947	1,235	1,399	1,854	2,406	2,026	1,378	522	136	76	109	259	521	11,921
1948	996	1,174	1,751	1,554	1,428	1,308	438	150	88	151	395	781	10,214
1949	1,389	2,059	3,058	2,589	1,488	1,048	253	82	79	125	226	470	12,866
1950	936	1,026	1,244	935	1,079	884	349	144	79	260	445	592	7,973
1951	1,427	1,545	1,951	2,200	1,964	1,284	523	142	93	148	198	374	11,849
1952	915	963	1,154	1,393	1,227	1,158	478	84	44	113	331	1,052	8,912
1953	1,747	1,796	2,625	2,411	1,877	1,052	543	110	67	124	160	430	12,942
1954	726	1,101	1,769	1,599	1,035	988	424	147	76	105	281	524	8,775
1955	1,044	1,247	2,023	1,903	1,653	1,247	372	107	75	94	224	487	10,476
1956	989	1,639	1,991	1,905	1,846	1,381	451	176	83	150	272	550	11,433
1957	798	902	1,316	1,443	1,102	705	272	119	106	72	115	337	7,287
1958	677	885	1,142	1,590	1,355	1,198	565	136	89	115	174	477	8,403
1959	741	1,034	1,401	1,316	1,056	882	506	136	80	62	149	368	7,731
1960	683	993	1,205	1,485	1,349	1,067	369	173	95	113	175	335	8,042
1961	619	691	1,022	858	953	913	332	127	82	62	147	384	6,190
1962	711	928	1,152	1,134	1,277	961	509	173	170	193	290	415	7,913
1963	707	724	1,105	1,065	938	755	442	170	91	150	219	317	6,683
1964	561	631	829	857	955	808	398	111	82	147	276	528	6,183
1965	746	889	1,274	1,164	1,024	863	436	270	156	139	156	306	7,423
1966	362	438	624	543	642	659	286	86	43	68	168	253	4,172
1967	526	601	809	759	950	1,088	452	198	82	72	154	206	5,897
1968	316	569	549	671	736	659	287	132	51	85	79	133	4,267
1969	177	210	372	562	623	626	296	142	82	96	166	288	3,640
1970	416	459	576	1,042	873	704	366	137	58	134	71	142	4,978
1971	211	331	471	639	569	718	391	123	72	63	86	141	3,815
1972	320	463	690	847	1,121	1,048							

Mumps—New York City

Year	Jan	Feb	Mar	Apr	May	June	July	Aug	Sept	Oct	Nov	Dec	Total
1928	124	132	193	144	195	105	40	20	42	58	170	197	1,420
1929	329	417	968	989	1,264	1,074	352	118	69	91	182	233	6,086
1930	406	550	853	835	787	518	132	66	57	77	83	138	4,502
1931	116	177	288	299	315	329	143	50	59	74	110	180	2,140
1932	361	556	687	821	799	901	478	212	153	207	386	576	6,137
1933	807	923	1,604	1,475	1,123	662	253	116	70	107	147	155	7,442
1934	266	249	406	452	547	472	213	136	114	137	276	341	3,609

Mumps—New York City

Year	Jan	Feb	Mar	Apr	May	June	July	Aug	Sept	Oct	Nov	Dec	Tota
1935	688	935	1,850	1,938	1,806	1,030	472	151	123	133	195	255	9,576
1936	357	386	543	645	668	659	276	142	93	82	122	225	4,198
1937	315	360	777	1,070	1,115	1,200	550	279	154	170	290	488	6,768
1938	764	990	1,695	1,738	1,295	1,026	345	178	85	115	122	179	8,532
1939	266	326	454	510	555	531	358	177	131	189	262	398	4,157
1940	662	733	941	1,229	1,485	1,406	959	436	233	221	391	696	9,392
1941	930	899	1,261	1,245	847	626	348	168	132	142	201	399	7,198
1942	569	651	1,024	1,272	1,258	960	445	201	138	117	162	272	7,069
1943	395	552	926	1,062	1,070	846	355	172	109	94	184	245	6,010
1944	324	433	752	845	859	671	275	134	75	104	161	237	4,870
1945	330	467	707	667	711	793	429	211	140	207	251	289	5,202
1946	433	468	799	1,138	1,048	710	438	178	114	118	175	206	5,825
1947	412	477	641	814	846	883	591	208	177	221	311	680	6,261
1948	1,105	1,490	1,956	1,713	1,291	1,053	366	153	95	117	130	183	9,652
1949	312	417	596	554	510	527	290	162	96	97	191	353	4,105
1950	604	767	1,116	1,103	1,330	1,342	670	319	167	176	261	363	8,218
1951	616	634	808	902	1,003	833	475	243	167	152	182	279	6,294
1952	520	615	745	838	787	827	570	254	182	185	268	530	6,321
1953	813	785	1,266	1,495	1,659	1,532	1,071	458	265	230	299	634	10,507
1954	802	945	1,220	1,088	792	707	420	199	134	123	176	284	6,890
1955	371	491	851	923	882	945	561	324	205	225	470	657	6,905
1956	1,066	1,371	1,674	1,844	1,819	1,469	769	393	176	240	299	375	11,495
1957	584	490	702	769	765	601	389	217	165	120	137	204	5,143
1958	288	348	515	774	751	711	555	280	182	160	255	565	5,384
1959	753	890	1,183	1,117	983	988	573	278	219	171	270	415	7,840
1960	510	587	721	754	699	667	355	230	145	129	235	263	5,295
1961	361	350	551	488	631	717	452	293	165	180	230	276	4,694
1962	435	487	563	608	803	615	554	297	166	213	303	517	5,561
1963	703	735	1,037	1,078	954	729	552	321	166	177	216	310	6,978
1964	342	268	304	280	286	280	213	143	101	117	132	277	2,743
1965	280	424	668	635	701	926	706	493	309	259	324	541	6,266
1966	569	672	1,020	848	754	765	437	255	166	130	165	166	5,947
1967	235	261	390	424	500	440	294	167	126	124	162	182	3,305
1968	274	349	420	607	572	496	415	240	158	124	135	100	3,890
1969	164	148	298	506	567	639	447	283	158	223	212	333	3,978
1970	330	281	354	527	463	429	300	147	104	90	77	122	3,224
1971	146	149	248	300	223	294	235	110	88	68	88	132	2,081
1972	149	157	219	221	264	298							

Measles—Baltimore

Year	Jan	Feb	Mar	Apr	May	June	July	Aug	Sept	Oct	Nov	Dec	Total
1928	1,158	2,371	3,752	2,746	1,792	320	45	10	5	4	10	16	12,229
1929	12	10	15	17	16	21	9	2	4	6	23	10	145
1930	12	21	21	92	146	52	17	5	6	3	13	63	451
1931	708	1,633	4,058	4,537	2,973	963	102	11	8	5	7	14	15,019
1932	11	9	18	24	23	29	9	10	6	3	8	15	165
1933	12	8	14	16	28	9	8	5	1	7	7	13	128
1934	95	916	3,002	6,018	6,872	1,516	140	9	6	6	24	8	18,612
1935	28	46	78	110	151	81	10	3	3	5	7	11	533
1936	41	98	299	720	1,321	917	290	55	20	17	134	449	4,361
1937	914	1,419	3,059	2,229	1,134	365	65	8	7	10	11	6	9,227
1938	24	14	51	70	122	108	37	19	25	89	157	403	1,119

APPENDIX 1—*Continued*

Measles—Baltimore

Year	Jan	Feb	Mar	Apr	May	June	July	Aug	Sept	Oct	Nov	Dec	Total
1939	2,412	4,080	3,195	1,368	585	147	13	2	11	7	7	6	11,833
1940	7	6	8	2	11	9	4	8	4	11	11	7	88
1941	27	75	266	570	894	1,168	572	118	38	41	171	518	4,458
1942	767	955	1,738	1,684	916	280	57	17	14	6	5	6	6,445
1943	25	60	188	418	575	534	196	57	7	16	33	104	2,213
1944	507	2,076	3,885	2,477	1,136	190	31	3	3	6	4	6	10,324
1945	23	16	28	37	38	19	12	4	6	6	5	12	206
1946	102	442	1,447	1,801	2,108	1,590	542	51	9	7	14	23	8,136
1947	23	23	26	44	66	52	18	8	4	3	2	5	274
1948	9	33	144	570	2,251	3,062	945	201	60	59	373	1,236	8,943
1949	2,190	3,618	3,680	1,095	267	97	13	7	3	6	22	33	11,031
1950	25	24	41	46	103	72	31	5	4	1	5	0	357
1951	13	25	138	239	484	831	644	237	82	211	573	899	4,376
1952	1,403	1,500	1,008	715	283	131	26	10	3	10	20	17	5,126
1953	9	11	52	124	232	151	57	26	18	34	113	237	1,064
1954	485	1,008	1,910	1,511	560	207	53	10	6	7	3	4	5,764
1955	5	30	87	68	34	55	26	16	16	21	103	464	925
1956	1,296	1,493	1,130	550	255	94	52	21	19	11	11	11	4,943
1957	35	13	12	27	68	78	112	55	37	69	395	858	1,759
1958	1,143	895	573	381	278	144	96	39	14	21	60	83	3,727
1959	134	87	106	153	192	153	102	29	32	22	27	91	1,128
1960	221	327	532	410	328	163	68	26	11	17	23	56	2,182
1961	128	173	272	353	392	324	108	68	19	44	83	125	2,089
1962	280	320	256	260	265	162	58	20	8	4	14	10	1,657
1963	45	27	69	134	240	236	134	90	34	61	107	277	1,454
1964	413	332	413	348	171	88	17	3	9	5	10	20	1,829
1965	109	47	142	144	107	83	81	41	7	10	32	141	944
1966	225	287	257	154	130	73	20	6	5	7	2	4	1,170
1967	9	7	14	12	9	9	7	7	5	5	3	5	92
1968	7	0	4	5	9	6	13	1	6	3	1	1	56
1969	1	1	2	11	1	25	3	10	0	0	7	16	77
1970	9	12	0	394	167	115	22	3	0	0	8	4	734
1971	2	3	0	0	7	2	1	0	0	0	1	1	17
1972	0	0	1	1	0	0							

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* During the 1941 disease year, over 85,500 cases of measles—more than twice the number of any other year—were reported in New York City. All boroughs showed proportional increases, the relative percentages being the same as in other high years, and, beginning in the fall, all months showed exceptionally high totals. The reasons for the unusual outbreak are not clear. After two successive low years a high year would be expected in 1941. Unlike other years, several newspaper articles about the outbreak began appearing in February of 1941 and perhaps more than the usual one in five to seven cases were reported that spring. For measles in New York City, the 1941 disease year is excluded from the study